Practical implementation of a harmonic conductance model in thermal simulation software

Tomasz Kornicki [1]

[1] T.Kornicki Dienstleistungen in EDV&IT in Vienna, Austria

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SUMMARY:
The concept of harmonic thermal conductances is very well suited for modelling dynamic thermal behaviour under periodic boundary conditions. A method for implementing such a model in simulation software is proposed and compared to simplified methods currently employed in international norms and standards. The practicality and efficiency of the harmonic approach is demonstrated using an appropriate simulation software package ("AnTherm"), which allows the three-dimensional, time-dependant visualisation of simulation results. In conclusion, arguments for specific future developments are presented, including the potential for model translation into 4D-enabled virtual environments.

1. Introduction

Over more than 6 decades there was research done extensively in the field of dynamic simulation of time-dependant, transient thermal building behaviour. Recently, the increasing importance of practical implementation has been noted, as for example in the works of Rantala (2005) or Nackler (2010).

Especially two compelling, related approaches shall receive reader's attention:

- time-step based, as that of Hagentoft (1988), initially founded for the purpose of assessing heat flows in building components in connection with the ground (the foundation of EN ISO 13370),
- and direct harmonic (periodic) method based, as that finally consolidated by Krec (1993-2000).

Both have led to the development of specialised numerical software for assessing thermal bridging effects. Focusing on the second approach, its straightforwardness and conformity with current standards, as well as practical application methods will be shown.

2. The simulation model of thermal conductances

The concept of thermal conductances and thermal weighting factors (g-values) was initially presented in 1987 in the book "Wärmebrücken" (Heindl 1987) and led to practical software implementations related to thermal bridges while assessing the risk of surface condensation (Krec 1989). It then gave rise to the hope that thermal bridges will lose their threat someday soon. Initial and practical implementations suitable for executing two-dimensional calculations on personal computers were already available at that point. They were developed on behalf of the Bureau of Applied Mathematics in Vienna and lead to the creation of the two- and three dimensional simulation program „Waebru," from today's perspective a predecessor to „AnTherm."

This concept can be easily extended to describe dynamic periodic processes. When this is done, the thermal conductances are complex numbers and are referred to as "harmonic thermal conductances." It is important to note that the term "thermal conductance" is also occasionally referred in standards and publications as "thermal transmittance" or "thermal coupling coefficient."

The software implementation available in 1993 covered the generalisations of earlier concepts towards dynamic heat transfer mechanisms involving thermal storage within buildings, as described by Krec

(1993a). It was subsequently extended by the same author to include consideration of power sources (Krec 1993b). By using case studies of components in connection with the ground, Krec demonstrated the efficiency and practicality of these early developments in everyday use (Krec 1993c and 1994).

Dynamic numerical indicators, such as “amplitudes” and “phase lags,” for two- and three-dimensional problems leading to equation systems of anywhere from hundreds of thousand to several million equations, are derived from the concept of harmonic thermal conductance. The latter can be calculated directly, i.e. neither with the need to know nor to supply any boundary conditions.

2.1 Harmonic conductance model

Any time dependant periodic entity can be developed by a Fourier series:

$$f(t) = \sum_{n=-\infty}^{+\infty} \hat{f}_n \cdot e^{-j \cdot \omega_n \cdot t} \tag{1}$$

$$\omega_n = n \cdot \frac{2 \cdot \pi}{T} \tag{2}$$

where:	n	index of the harmonic
	f_n	amplitude represented as complex number
	T	period length (year, day, etc.)
	ω	angular oscillation frequency
	t	time

The heat flow (transmission) equation:

$$\hat{\Phi}_i^h = -\iint_{\Re_i} (\lambda \cdot \mathrm{grad}\, \hat{T}^h) \cdot d\vec{a} \tag{3}$$

where:	Φ_i	complex amplitude of the heat loss for the h-th harmonic
	R_i	boundary surface of the space i
	λ	the thermal conductivity of the material
	da	the surface element oriented towards the space
	T^h	complex amplitude of the h-th harmonic of the temperature oscillation

Base solutions:

$$\hat{T}^h(x, y, z) = \sum_j \tilde{g}_j^h(x, y, z) \cdot \hat{\Theta}_j^h \tag{4}$$

where:	T^h	complex amplitude of the h-th harmonic of the temperature oscillation in or at the component
	j	space number
	g_j	the base solution (complex temperature weighting factor) at location (x,y,z)
	θ_j	the complex amplitude of air temperature

By combining the above two equations, we get

$$\hat{\Phi}_i^h = -\sum_j \hat{\Theta}_j^h \cdot \iint_{\Re_i} (\lambda \cdot \mathrm{grad}\, \tilde{g}_j^h) \cdot d\vec{a} \tag{5}$$

and thus are now able to provide a clear definition of the harmonic thermal conductance which indicates the degree of thermal coupling:

$$\boxed{\tilde{L}^{h}_{i,j} = \iint_{\Re_i} (\lambda \cdot \operatorname{grad} \tilde{g}^{h}_{j}) \cdot d\vec{a}} \tag{6}$$

The equation (5) can be now written as:

$$\hat{\Phi}^{h}_{i} = -\sum_{j} \tilde{L}^{h}_{i,j} \cdot \hat{\Theta}^{h}_{j} \tag{7}$$

where: $L^{h}_{i,j}$ the complex thermal coupling coefficient (harmonic thermal conductance) for h-th harmonic

i,j indices of boundary conditions (spaces)

2.2 Periodic boundary conditions

To compute the yearly distribution of heat losses through the ground, the rules of EN ISO 10211 should be applied to model the complete ground connected structure (ground slab, walls, cellars, etc.) in three dimensions. A harmonic periodic calculation for the period of one year yields the steady state matrix of thermal coupling coefficients $\{L_{i,j}\}$ and the matrix of harmonic thermal coupling coefficients $\{\tilde{L}_{i,j}\}$.

The (yearly) mean value of heat losses $\{\bar{\Phi}_i\}$ of a space i can be determined by applying the simple calculation

$$\bar{\Phi}_i = -\sum_{j=1}^{N} L_{i,j} \cdot \bar{\Theta}_j \tag{8}$$

where: $L_{i,j}$ the element of the matrix of steady state thermal coupling coefficients

$\bar{\Theta}_j$ the mean value of air temperature in the space j

The summation is to be executed through all N spaces connected to the model – this also includes the space i .

The (yearly) mean value $\bar{\Phi}_i$ shall be superposed by the yearly oscillation, of which the amplitude $\hat{\Phi}_i$ can by calculated by analogy to equation (8) by

$$\hat{\Phi}_i = -\sum_{j=1}^{N} \tilde{L}_{i,j} \cdot \hat{\Theta}_j \ . \tag{9}$$

$\tilde{L}_{i,j}$ are complex numbers and elements of the matrix of harmonic thermal coupling coefficients and $\hat{\Theta}_j$ are complex amplitudes of the yearly oscillation of air temperature in the space j.

The time dependant distribution of heat losses during the year can be determined by a Fourier synthesis of the results for the entire building.

2.3 Use in Norms and Standards

These theoretical concepts, among others, have been incorporated in the “thermal bridge standard” EN ISO 12011. Additional standards (e.g., EN ISO 6946, 13370, etc.) made direct use of these concepts during the late 1990s and thus provide strong conformity of the current implementation in AnTherm with these standards and recommendations.

Further generalisation of the concept to describe dynamic transmission processes involving heat capacity consideration under periodic boundary conditions has also been established within EN ISO 13768, which can thus be seen as a further step in the implementation of the thermal conductance concept.

The parameters and indicators defined in these standards have also been incorporated in various other calculation and building simulation policies and systems, such EBPD, EnEV or OIB, energy certification, and energy passport or PHPP (Feist 2007), and can be calculated efficiently with AnTherm.

2.4 Relation to EN ISO 13370 and PHPP

For the special consideration of a “single zone model,” i.e. where all interior spaces of the building share the same temperature (1 zone) and with the exterior space also considered isotherm, the equation (8) can be reduced to the form of

$$\bar{\Phi}_1 = -L_{1,0} \cdot \bar{\Theta}_0 - L_{1,1} \cdot \bar{\Theta}_1 = L_{1,0} \cdot (\bar{\Theta}_1 - \bar{\Theta}_0)\,. \qquad (10)$$

In this equation (10) the interior is indexed by 1 and the exterior by the index 0. By analogy, the equation (9) applied for the special case of 2-spaces-only receives the form of

$$\hat{\Phi}_1 = -\tilde{L}_{1,0} \cdot \hat{\Theta}_0 - \tilde{L}_{1,1} \cdot \hat{\Theta}_1\,. \qquad (11)$$

Applying the Fourier synthesis yields the following form:

$$\Theta_1(t) = L_{1,0} \cdot (\bar{\Theta}_1 - \bar{\Theta}_0) + \left|\tilde{L}_{1,0}\right| \cdot \left|\hat{\Theta}_0\right| \cdot \cos(\omega \cdot t + \varphi) + \left|\tilde{L}_{1,1}\right| \cdot \left|\hat{\Theta}_1\right| \cdot \cos(\omega \cdot t + \psi) \qquad (12)$$

(intentionally expanded and written in real numbers).
For the oscillation frequency $\omega = \frac{2 \cdot \pi}{P}$, the period length *P* is set depending on the purpose of evaluation (daily evaluation: *P* = 365, monthly evaluation *P* = 12, etc.).

By comparison with the method shown in EN ISO 13370:2008-4 to calculate heat losses in a month *m*

$$\Theta_m = H_g \cdot (\bar{\Theta}_i - \bar{\Theta}_e) + H_{pe} \cdot \hat{\Theta}_e \cdot \cos(2 \cdot \pi \cdot \frac{m - \tau + \alpha}{12}) - H_{pi} \cdot \hat{\Theta}_i \cdot \cos(2 \cdot \pi \cdot \frac{m - \tau - \beta}{12}) \quad (13)$$

one can easily recognize that the indicators defined there also show the following relation to harmonic thermal coupling coefficients (harmonic conductances):

$$H_g = L_{1,0} \qquad H_{pe} = \left|\tilde{L}_{1,0}\right| \qquad H_{pi} = \left|\tilde{L}_{1,1}\right| \qquad (14)$$

and phase lags φ und ψ in (12) are normalised to the period length in α and β in (13).

Hence, the results directly provided by AnTherm as matrices of steady state coupling coefficients and harmonic thermal coefficients calculated for the transient state of a one year period are directly translatable to the indicators H_g, H_{pe}, H_{pi}, α and β defined by EN ISO 13370.

When PHPP (Feist 2007) is used, its approach neglects interior air temperature changes during the period ($\hat{\Theta}_i = 0$) and by that, only values of H_g, H_{pe} and α are used therein.

3. AnTherm simulation of thermal behaviour

AnTherm is a powerful innovative software application for building physics that integrates new visualisation capabilities, which have their origins in supercomputing and in the scientific visualisation of large sets of physical data, in the daily life of a civil engineer. Features unavailable to building physicists only a short time ago (due to the high complexity and unacceptable learning costs involved) are provided in AnTherm in an intuitively graspable way. It employs extensive automation of the numerical models used, meaning that calculation results are (almost) immediately assessable and high quality visualisations can be transferred directly into the reports.

3.1 Implementation requirements

Thermal conductances (i.e. the matrix of thermal coupling) shall be seen as characteristic indicators of a building construction and hence are independent of boundary conditions. Therefore, the matrix of thermal conductances must be calculated directly without the need to specify any boundary conditions.

Presentation of results should conform to standards and be easily transferable to other simulation systems by a few simple manual calculations at most. Depending on the task set, standards such as EN ISO 10211, 10077, 6946, 13370, 13788, 13786 etc. define the requirements to be fulfilled and the parameters to be used very precisely. Conformity to such standards can be easily verified by the users themselves for each and every simulation.

Immersive 2D and 3D visualisations are required to efficiently support evaluation of two- and three dimensional problems through rendered representations of thermal processes within the construction. Such images can be easily transferred into external reports or for further processing.

The numerical stability of the method used must suffice for most tasks without the need of any specialised user actions. This must also be assured in combination with very detailed components, such as thin vapour barriers, sealing membranes, reinforcements etc., whose dimensions can be far below 0.01mm, together with substantially more voluminous constructions and entire complex buildings (e.g., those in contact with the ground, of more than 100m extent, with multiple interior spaces, etc.).

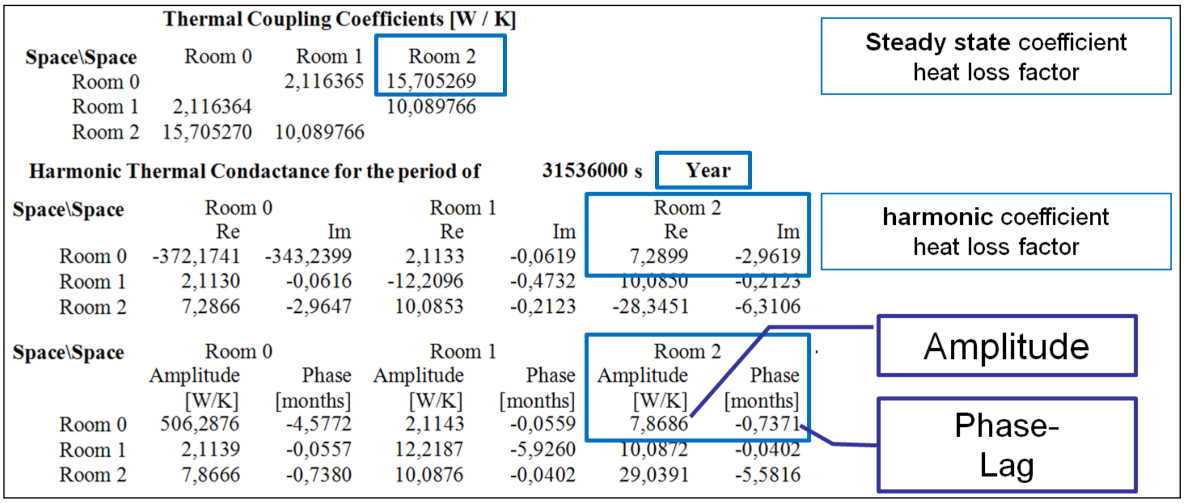
The integrated implementation allows effective execution after a short learning period and ensures conscious control of the tool without specialised knowledge. The tool should integrate itself into the everyday work process of an engineer, architect, building specialist or assessment expert, even if it is used only occasionally. Computation may take only seconds or, at most, a few minutes when executed on typical office hardware.

When used in the classroom, it should support and facilitate insightful learning. The application of advanced simulation and visualisation techniques significantly enhances the enjoyment of exploring the thermal performance of buildings and encourages the user to create optimised component variations.

3.2 Numerical solution

The linear system of equations is derived from the heat transfer equation for the harmonic periodic model in the discrete space and represented by complex numbers. This leads to a linear system of several hundred thousand to a few millions of equations. The solution vectors of base solutions for every period length contain the weighting factors (g-values) of respective cells of the discrete finite difference model and are complex numbers as well. When the special case of the steady state is considered (i.e. under invariant, constant boundary conditions), the imaginary parts of the equation system and of the solution vector disappear. The symmetry and significantly dominant diagonal of the equation matrix ensure the property “A” of the Jacobi matrix used for the iterative over-relaxation (following Gauß-Seidl with varied relaxation factor). Hence the convergence of the method is assured (Björck 1974).

The harmonic thermal coupling coefficients and the phase lag between the oscillation of the temperature and the resulting variation of heat losses are calculated directly and independently of any boundary conditions. They are represented as complex numbers and (alternatively) given by amplitude and phase as shown in Figure 1. These numerical results are available for immediate calculation of several derived indicators, such as those defined by EN ISO 13768.



Thermal Coupling Coefficients [W / K]

Space\Space	Room 0	Room 1	Room 2
Room 0		2,116365	15,705269
Room 1	2,116364		10,089766
Room 2	15,705270	10,089766	

Harmonic Thermal Condactance for the period of **31536000 s** **Year**

Space\Space	Room 0		Room 1		Room 2	
	Re	Im	Re	Im	Re	Im
Room 0	-372,1741	-343,2399	2,1133	-0,0619	7,2899	-2,9619
Room 1	2,1130	-0,0616	-12,2096	-0,4732	10,0850	-0,2123
Room 2	7,2866	-2,9647	10,0853	-0,2123	-28,3451	-6,3106

Space\Space	Room 0		Room 1		Room 2	
	Amplitude [W/K]	Phase [months]	Amplitude [W/K]	Phase [months]	Amplitude [W/K]	Phase [months]
Room 0	506,2876	-4,5772	2,1143	-0,0559	7,8686	-0,7371
Room 1	2,1139	-0,0557	12,2187	-5,9260	10,0872	-0,0402
Room 2	7,8666	-0,7380	10,0876	-0,0402	29,0391	-5,5816

Steady state coefficient
heat loss factor

harmonic coefficient
heat loss factor

Amplitude

Phase-
Lag

FIG 1. Example of the Matrix of Thermal Coupling Coefficients (thermal conductance matrix) contained within reports created in AnTherm

3.3 Visualisation of simulation results

Interactive 2D and 3D visualizations of thermal processes support a better understanding of them and can easily be used as planning or component optimisation support, as well as in research or education. “If houses could talk …” they would tell us endless stories of avoidable building defects and planning errors resulting from the blind fear of the – from today's perspective no longer existent - complexity of specialised investigation. Thanks to the state of the art computer-aided visualisation methods, heat flow processes are easily “made perceivable” and integrated in the highest quality to the desktops of building physics experts. Thermal bridges can be explored and examined in an almost playful, enjoyable manner.

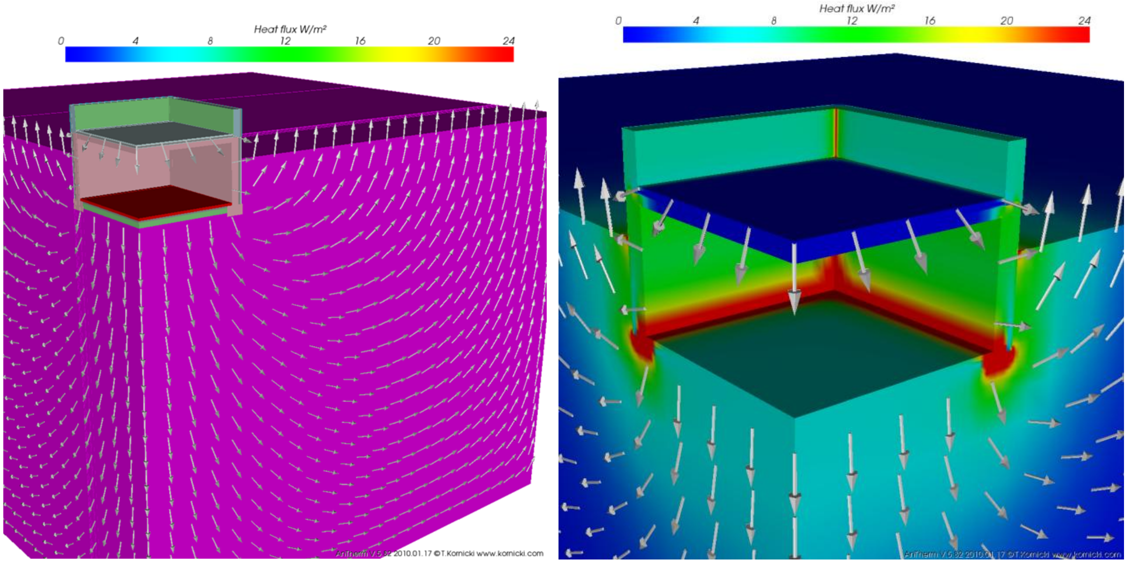


FIG 2. Visualisation of heat flows through a building component in extensive ground contact to support the assessment, localisation, and identification of thermal weaknesses in the construction.

The evaluations are highly interactive and, since each analysis and visualisation algorithm is processed in fractions of a second in a typical case, give the impression of a real-time simulation.

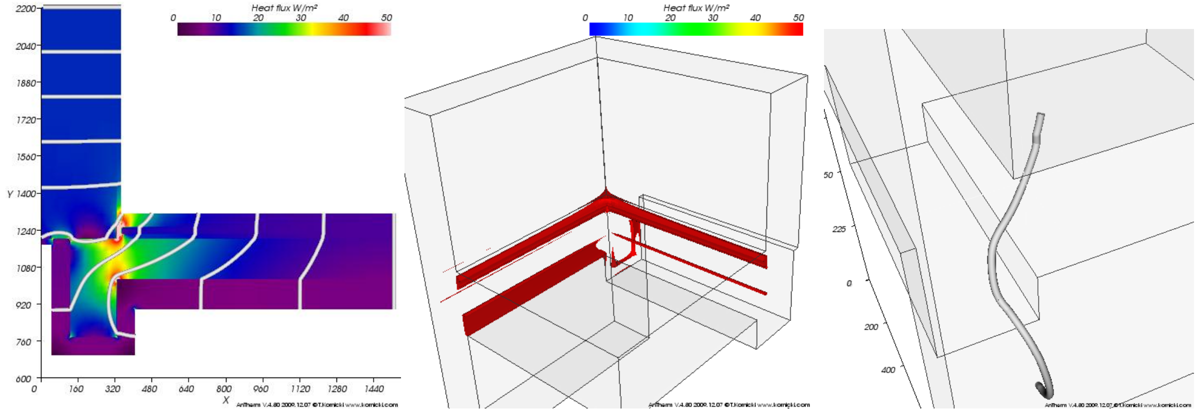


FIG 3. Two-dimensional visualisation of heat flow with streamlines and colourised heat flux values (left), three-dimensional iso-surface visualisation of specific heat flux values (center), and a heat flow trace with a streamline dynamically generated from the three-dimensional heat flux vector field (right).

4. Conclusions

The effectiveness of the thermal conductance and base solutions concept has been shown by example using the software package AnTherm. The implementation integrates seamlessly into engineering processes and provides not just the direct calculation of component-related properties in conformance with international standards, but also a productive method of facilitating a deeper understanding of the heat transfer mechanisms prevailing in building structures.

The need for further development is demonstrated, for example, by the following conclusions drawn from this work:

- Transformation of the harmonic (periodic) problem from the complex number equation system to one represented by real numbers could lead to the equation matrix populating only two quadrants of the Jacobi matrix. For this matrix, the proof of property “A” shall be provided (as already provided by the symmetry and diagonal dominance of the steady state problem equation matrix), thus making the Gauß-Seidl over-relaxation a numerically stable and more time-efficient method for solving this type of transient problem.
- Further extension of the visualisation techniques could effectively employ advanced volume-rendering methods to provide an optimal view of heat flux distribution while three-dimensional models are analysed, thus allowing for an even easier identification of thermally weak spots in building constructions. Such methods of visualising complex scalar and vector fields are already well known in the community from other application fields of visual computing, such as medical diagnostics.
- Supplemental extensions of calculation, evaluation, and visualisation methods should aim at deployment on massively parallel systems, while continuing to respect the currently typical equipment of engineering bureaus.
- The 4D visualisation of time-dependant simulation results in immersive virtual environments is within reach.

5. Acknowledgements

Successful implementation of the AnTherm project (“anthem” = hymn) would not have been possible without the works of Dr. Walter Heindl (†1994) and Dr. Klaus Krec, two of the “fathers” of the thermal conductance and base solutions concept. Thanks also to Dr. Margit Rudy for her editorial support.

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